A NOTE ON LIE GROUPS AND LIE ALGEBRAS

DEBAPRIYA BISWAS

Department of Mathematics, Indian Institute of Technology, Kharagpur, India

Abstract: The theory of Lie groups, Lie algebras and their applications, form a crucial part of mathematics. Since Second World War, it has been the focus of an increasing research effort, and is now seen to have applications in various mathematical areas, including classical, differential and algebraic geometry, topology, ordinary and partial differential equations, complex analysis (one and several variables), groups and ring theory, number theory, and physics from classical to quantum and relativistic.

It is impossible in a short space to convey the full scope of the subject, but we will cite some examples. The essential phenomenon of Lie theory, is that one may associate in a natural way to a Lie group G, its Lie algebra g. The Lie algebra g is first of all a vector space and secondly is endowed with a bilinear non-associative product called the Lie bracket or commutator and is usually denoted by [,].

Amazingly, the group G is almost completely determined by g and its Lie bracket. Thus for many purposes one can replace G with g. Since G is a complicated non-linear object and g is just a vector space, it is usually vastly simpler to work with g. Otherwise, intractable computations may become straightforward linear algebra. This is one source of the power of Lie theory.

The basic object mediating between Lie groups and Lie algebras, is the one-parameter group. We then define and give examples of matrix groups, the class of Lie groups, considered in this paper. Then we define Lie algebras and show that every matrix group can be associated to a Lie algebra which is related to its group in a close and precise way and finally give an elementary proof of Campbell-Baker-Hausdorff theorem due to Eichler.