FUZZY SETS AND ROUGH SETS: A CATEGORY THEORETIC COMPARISON

MIHIR KUMAR CHAKRABORTY

Professor (Retired) of Pure Mathematics, University of Calcutta, India

Visiting Professor, Jadavpur University, Kolkata, India

Abstract: Our talk is a purely mathematical one dealing with a common possible foundation of Fuzzy Set Theory and Rough Set Theory. It begins with a generalization of Obtulowicz's paper, Rough sets and Heyting algebra valued sets, published in [4]. Obtulowicz proposes a special subcategory of Higgs' Category and claims that a slightly modified version of Pawlak Rough Sets form the objects of this subcategory.

Basically, we follow the following steps during this talk.

Section 1. Introducing Obtulowicz's approach.

Section 2. Extension of the above mentioned approach by using rough membership function.

Section 3. Presentation of logical consequences and limitations of Obtulowicz's category and its extension proposed by us in Section 2.

Section 4. Modification of the approach proposed in Section 2 in the direction of mitigating the above mentioned limitations.

Section 5. Relative location of these newly proposed categories and logics developed thereby with respect to the existing categories.

Section 6. Conclusion: Fuzzy Sets and Rough Sets – a possible bridge between them.

We give below a little more detail of the contents.

Section 1. A 4-valued Heyting algebra $L4 = \langle 0, 1, 2, 3 \rangle$ with usual ordering is taken. A category is defined where objects are pairs of the form $\langle A, E \rangle$, A being a non-empty set and E, a fuzzy equality on A, that is a mapping from $A \times A$ to L satisfying the conditions

$$0 < E(x, x),$$

$$E(x, y) \le E(x, x),$$

$$E(x, y) = E(y, x),$$

$$E(x, y) \wedge E(y, z) \le E(x, z).$$

Morphisms are those of Higgs' category with certain other conditions called "roughness conditions". Obtulowicz, through two representation theorems established that Pawlak Rough Sets can be validly identified to be the objects of this special kind of Higgs' category.

Section 2. We extend the 4-valued Heyting algebra to $n+1$-valued one viz. $Ln = \langle 0, 1, 2, \ldots, n \rangle$, and obtain the corresponding category. Objects of the corresponding category may be interpreted as Pawlak Rough Sets but in terms of rough membership functions. In fact, the values from $2, \ldots, n-1$ are interpreted as the rough membership values of the elements in the boundary of a set in an approximation space. One could see that this generalization is non-trivial.

Section 3. The $n+1$ valued objects may also be considered as the interpretation of a well-formed formula in an $n+1$ valued logic. In fact, an $n+1$ valued intuitionistic logic will naturally emerge. But in rough set context, intuitionistic logic is not, in our opinion, the appropriate logic.

Section 4. So, we first modify the logic that we think appropriate and then go back to the category. The earlier category is now modified so as to fit in best with the intuitive understanding of rough sets.

Section 5. These categories are compared with the existing well-known categories pertaining to rough sets and fuzzy sets.

Section 6. This work will as a consequence point at some relationship between Fuzzy Sets and Pawlak Rough Sets.

References

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