MODELING STOCHASTIC DEPENDENCE RELATED TO SOME RELIABILITY AND BIOMEDICAL PROBLEMS

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Abstract: Our goal is to construct the probabilistic models mainly for the estimation of random (residual) life-time of a human that is subjected to some, harmful stresses during some (also random) period of time. More specifically, we consider a relatively new kind of stochastic dependence of the life-time, say T on the random amounts of stresses, say $X_1, X_2, ..., X_m$. The obtained stochastic models are shown to find application to both biomedical and reliability problems.

Two basic situations are considered. In the first situation, the biological or technical (reliability) object is free from any 'special' burdens or stresses. It is assumed that in this situation the baseline hazard rate of T is represented by a nonnegative time function $h_0(t)$ or, equivalently, by the corresponding baseline probability distribution $F_0(t; \theta)$ which depends on a fixed scalar or a vector parameter θ. In the second situation, a "statistically the same" as before, bio-object is subjected to some additional extra stresses, numerically expressed by given numbers $x_1, x_2, ..., x_m$. These stresses impact a human (residual) or other object's life-time T.

The impact, we define, may be thought of as "indirect". By this, we mean that any nonzero realizations $x_1, x_2, ..., x_m$ of the random stresses $X_1, X_2, ..., X_m$ do not strictly determine the value t of the patient's life-time T as a result of an "algebraic" transformation, say $(x_1, x_2, ..., x_m) \to t$, even if (as it is typically done) we would take under consideration a random error, say ε, in the accuracy of determining the value of t of T. Since, besides the influence of the random stresses $X_1, X_2, ..., X_m$ on the human life-time T, the time considered here is the quantity that has its own randomness expressed by its baseline probability distribution $F_0(t; \theta)$. Therefore, we should rather deal with a kind of "composition" of the two sources of randomness. For this reason, we rather consider an influence of realized values of the stresses $x_1, x_2, ..., x_m$ on the considered life-time T "indirectly", through influencing its original (baseline) probability distribution as

$$(x_1, x_2, ..., x_m) \to F_0(t; \theta) \tag{1}$$

or, equivalently, by influencing its baseline hazard rate

$$(x_1, x_2, ..., x_m) \to h_0(t; \theta). \tag{2}$$

In the case considered here, we define an influence of all the combined stresses $x_1, x_2, ..., x_m$ on T's probability distribution $F_0(t; \theta)$ as an influence on this distribution's parameter θ. Symbolically, the "stresses joint action" is expressed by the diagram:

$$(x_1, x_2, ..., x_m) \to \theta. \tag{3}$$

Thus, any realization $(x_1, x_2, ..., x_m)$ of the random vector $(X_1, X_2, ..., X_m)$ determines a new value, say $\theta^* = \theta^*(x_1, x_2, ..., x_m)$ of the distribution's parameter (that initially was equal to θ). Only the assumption of continuity is imposed on the function $\theta^*(x_1, x_2, ..., x_m)$ of its m arguments.

This method enables us to determine numerous conditional probability distributions (and, correspondingly, conditional hazard rates) of the life-time T, given the random event $(X_1, X_2, ..., X_m) = (x_1, x_2, ..., x_m)$ which occurred. This conditional probability distribution's pattern can be expressed in the following compact form:

$$F(t \mid x_1, x_2, ..., x_m) = F_0(t; \theta(x_1, x_2, ..., x_m)). \tag{4}$$

One should admit that such a method defines a very general and relatively a new class of stochastic dependences. As one may expect, many probabilistic models satisfying (4) can be constructed and applied.